Review on Quadratic Forms

Outline of the Video Lecture

0'00" **Introduction**

1'45" **Tests to Know if a Matrix is Positive Definite**

Let us consider

$$A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

This matrix is positive definite if and only if one of the following statements is true (in which case, all of them are true.)

1. Both eigenvalues are positive
2. The 1 by 1 and 2 by 2 determinants are positive: $a > 0$ and $ac - b^2 > 0$
3. The pivots are positive: $a > 0$ and $(ac - b^2)/a > 0$
4. The function ${}^T x\, A\, x = ax_1^2 + 2bx_1x_2 + cx_2^2$ is positive

4'20" **Example**

Let us consider the matrix

$$\begin{bmatrix} 2 & 6 \\ 6 & ? \end{bmatrix}$$

What should be put in lieu of the question mark in order to have a definite positive matrix?

- He finds that 18 is the *borderline* between definite positive and not. He applies the tests.
- He tries 7 and sees what happens.

14'15" **Graph**

When he uses the example with ? = 7, the graph looks like a saddle.
When he uses the example with ? = 20, the graph looks like a bowl.

22'45" **Relation with the Derivatives**

Relations between derivatives and the graph. Comparison between the cases $N = 1$ and $N = 2$. (Note that 18.06 is the class which is being taught.)

27'00" **Canonical Form**

The relation between the canonical form and the pivots is shown in an example.

35'15" **Hessian Matrix**

Note that $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial y^2}$ and $\frac{\partial^2 f}{\partial x \partial y}$ are noted f_{xx}, f_{yy} and f_{xy}.

38'00" **Example in Dimension 3**